

Physical implementation of topologically decoherence-protected superconducting qubits

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We propose a scenario to physically implement a kind of topologically decoherence-protected qubit using superconducting devices coupled to a micro-wave cavity mode with unconventional geometric operations. It is shown that the two needed interactions for selective devices, which are required for implementing such protected qubits, as well as single-qubit gates, can be achieved by using the external magnetic flux. The easy combination of individual addressing with the many-device setup proposed in the system presents a distinct merit in comparison with the implementation of topologically protected qubits in a trapped-ion system.

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Physical implementation of quantum computers has attracted much attention as they are generally believed to be capable of solving diverse classes of hard problems. Systems suitable for hardware implementation of quantum computers should possess certain properties, such as relatively long coherent time, easy manipulation and good scalability. With highly developed fabrication techniques, superconducting quantum interference devices (SQUIDs) have shown their competence in implementing the qubits for scalable quantum computation [1]. Furthermore, the idea of placing the SQUIDs inside a cavity, *i.e.*, the circuit quantum electrodynamics, has been illustrated [2, 3, 4, 5] to have several practical advantages including strong coupling strength, immunity to noises, and suppression of spontaneous emission.

Decoherence and systematic errors always occur in real quantum systems and therefore stand in the way of physical implementation of quantum computers. Generally, larger systems are more sensitive to decoherence, which makes the scaling of quantum computers a great experimental challenge. Therefore, how to suppress the infamous decoherence effects is a main task for scalable quantum computation. A promising strategy for quantum computation in a fault-tolerant way is based on the topological idea [6], where gate operations depend only on global features of the control process, and are therefore largely insensitive to local noises. The Kitaev model [6] consists of a class of stabilizer operators associated with lattice on the torus, which can be put together to make up a Hamiltonian with local interactions. The four-fold degenerate ground state of the Hamiltonian coincides with the protected space of the stabilizer quantum code. Since all the excited states are separated from the ground states by an energy gap, the ground states are persistent to local perturbations. However, it is extremely difficult to directly implement this novel idea mainly because four-body interactions are notoriously hard to generate in experiments.

Very recently, Milman *et al.* [7] proposed a highly symmet-

rical Hamiltonian

$$H = -\hbar\chi_x \sum_{i=1}^M \left(\sum_{j=1}^M \sigma_{i,j}^x \right)^2 - \hbar\chi_y \sum_{j=1}^M \left(\sum_{i=1}^M \sigma_{i,j}^y \right)^2, \quad (1)$$

which involves only two-body interactions and can be understood as spins in a 2-dimensional (2D) lattice. Each spin was labeled by its position in the lattice with $\sigma_{i,j}^{x,y}$ denoting Pauli matrices of the spin situated at the intersection of the i th row and j th column. It has a two-fold degenerate ground state which may be considered as the two states in a qubit. These states are generated by non-local operators and thus are naturally protected against local sources of decoherence. However, comparing with a size-independent energy gap in the Kitaev Hamiltonian, the energy gap here is slightly dependent on the number of the involved spins. A nice feature of the model lies in that it is possible to directly implement the Hamiltonian (1) in a trapped-ion system, as it eliminates a main experimental constrain-the four-body interactions in the Kitaev model. In addition, the energy gap of Hamiltonian (1) remains large as the system size increases, and thus it provides more efficient protection than the nearest neighbor model [8] where energy gap decreases rapidly.

In this paper, we propose an intriguing scenario to implement the model Hamiltonian (1) with superconducting devices (serve as spins in [6, 7]) coupled to a micro-wave cavity mode with unconventional geometric operations. In trapped-ion systems, the individual addressing is difficult for large arrays ($N > 3$) since the distance between the ions in the center of the linear array gets smaller [9, 10]. This constrain makes the linear configuration proposed in Ref. [7] difficult to be experimentally implemented. Comparing with the above difficulty, a distinct feature of the present implementation is that the combination of individual addressing with a many-device setup is feasible. We will show that, by using the external magnetic flux as the effective switch tool, the interactions between selective devices can be introduced. Furthermore, single-qubit gates, which are required in the global state initialization, can also be achieved by the same technology. During the implementation of the operations, the only parameter needs to be tunable is the frequency of the external magnetic flux. This

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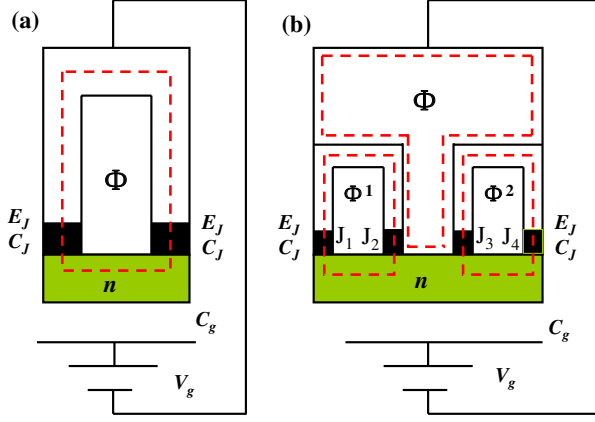


FIG. 1: (Color online) Schematic illustration of the superconducting device as the effective spin, the red dash line denote the integral path of the magnetic flux. (a) A single SQUID as an effective spin. (b) Device made of two SQUIDs with a common superconducting charge box. This more flexible design will introduce more control variables of the effective spin.

is a more efficient way of controlling the system dynamics as it is much easier for experimental realization. In achieving the implementation of the needed interactions and given the fact that selective coupling can be effectively controlled, it is possible to implement topologically protected qubits in this architecture. Moreover, the operations to achieve the above qubit states are based on unconventional geometric operations, which have been illustrated to have a high fidelity [11, 12, 13].

A SQUID as a superconducting charge qubit that usually addressed in the literature is shown in Fig. (1a) [1]. However, it would be clear that this simple configuration can not be used to achieve the coupling described by Eq. (1). To realize the desired Hamiltonian, we design a more flexible device, as shown in Fig. (1b), to serve as the effective spin. It consists of two SQUIDs with a common superconducting charge box, which has n excess Cooper-pair charges. Each SQUID is formed by two small identical Josephson junctions (JJs) with the capacitance C_J and Josephson coupling energy E_J , pierced by an external magnetic flux. A control gate voltage V_g is connected to the system via a gate capacitor C_g . The Hamiltonian of the system reads [1]

$$H = E_c(n - \bar{n})^2 - E_J \sum_{l=1}^4 \cos \varphi^l, \quad (2)$$

where n is the number operator of (excess) Cooper-pair charges on the box, $E_c = 2e^2/(C_g + 4C_J)$ is the charging energy, $\bar{n} = C_g V_g/2$ is the induced charge controlled by the gate voltage V_g , and φ^l ($l = 1, 2, 3, 4$) is the gauge-invariant phase difference between the two sides of the l th JJ denoted as J_l in Fig. (1b). The phase differences φ^l are determined from the flux quantization for three independent loops, that is, $\varphi^1 - \varphi^2 = 2\phi^1$, $\varphi^2 - \varphi^3 = 2\phi$, and $\varphi^3 - \varphi^4 = 2\phi^2$. Since we here focus on the charge regime, a convenient basis is formed by the charge states, parameterized by the number

of Cooper pairs n on the box with its conjugate $\varphi = \sum_l \varphi^l/4$ by the relation $[\varphi, n] = i$. At temperatures much lower than the charging energy and restricting the gate charge to the range of $\bar{n} \in [0, 1]$, only a pair of adjacent charge states $\{|0\rangle, |1\rangle\}$ on the island are relevant. Under the condition $\phi^1 = \phi^2 = 0$, the Hamiltonian (2) is then reduced to

$$H_s = -E_{ce}\sigma^z - 2E_\Phi\sigma^x, \quad (3)$$

where $E_{ce} = 2E_c(1 - 2\bar{n})$, $E_\Phi = E_J \cos(\pi\Phi/\phi_0) = E_J \cos \phi$ with $\phi_0 = hc/e$ being the normal flux quantum and $\phi = \pi\Phi/\phi_0$.

When the device is placed in a cavity, the gauge-invariant phase difference becomes $\varphi'_m = \varphi_m - \frac{2\pi}{\phi_0} \int_{l_m} \mathbf{A}_m \cdot d\mathbf{l}_m$, where \mathbf{A}_m is the vector potential in the same gauge of φ_m . \mathbf{A}_m may be divided into two parts $\mathbf{A}'_m + \mathbf{A}^\phi_m$, where the first and second terms arise from the electromagnetic field of the cavity normal modes and the external magnetic flux, respectively. For simplicity, we here assume that the cavity has only a single mode to play a role. Therefore, we have $\frac{2\pi}{\phi_0} \int_{l_m} \mathbf{A}_m \cdot d\mathbf{l}_m = \frac{2\pi}{\phi_0} \int_{l_m} \mathbf{A}^\phi_m \cdot d\mathbf{l}_m + 2g(a + a^\dagger)$, where $2g$ is the coupling constant between the junctions and the cavity, a (a^\dagger) is the annihilation (creation) operator for the cavity mode, with ω_c being its frequency, the closed path [the red dash line in Fig (1)] integral of the \mathbf{A}^ϕ gives rise to the magnetic flux Φ . Then the Hamiltonian (3) becomes

$$H_c = -E_{ce}\sigma_z - 2E_J \cos[\phi + g(a + a^\dagger)]\sigma_x. \quad (4)$$

We now show that the required single-qubit gates can be achieved by the dc magnetic flux. Generally speaking, since the coupling constant g is very small comparing with ϕ , we may expand the Hamiltonian (4) up to the first order of g . In the rotating frame with respect to $H_{dc} = -E_{ce}\sigma_z - 2E_\Phi\sigma_x + \hbar\omega_c(a^\dagger a + \frac{1}{2})$, the strength of the cavity-SQUID interaction term in Eq. (4) is proportion to $1/(\omega_c - \omega_q)$ with $\omega_q = 2E_c(1 - 2\bar{n})/\hbar$, which is an experimentally controllable parameter via the gate voltage V_g . In the present scheme, we choose $\omega_q = 0$, which corresponds to the degeneracy point. Given the facts that g is relatively small and the frequency difference $\omega_c - \omega_q$ is chosen to satisfy the large-detuning limit, the cavity mediated interaction can thus be safely neglected. In other words, the device and the cavity evolve independently in this case. The external flux is merely used to address separately the single-qubit rotations, while the evolution of the SQUID is governed by Hamiltonian (3).

The required interactions between the selected devices may be induced by using the ac magnetic flux with devices in Fig. (1b). If N devices are located within a single-mode cavity as shown in Fig. (2a), to a good approximation, the whole system can be considered as N two-level systems coupled to a quantum harmonic oscillator [3]. The Hamiltonian of the N such devices, placed in a single mode cavity, reads [5]

$$H = \sum_{j=1}^N \left[E_{ce}(n_j - \bar{n}_j)^2 - E_J \sum_l \cos \varphi_j^l \right], \quad (5)$$

where we have assumed $E_{ce,j} = E_{ce}$ and $E_{J,j}^l = E_J$. Here ϕ^1 and ϕ^2 are the dc magnetic flux, and we have neglected the

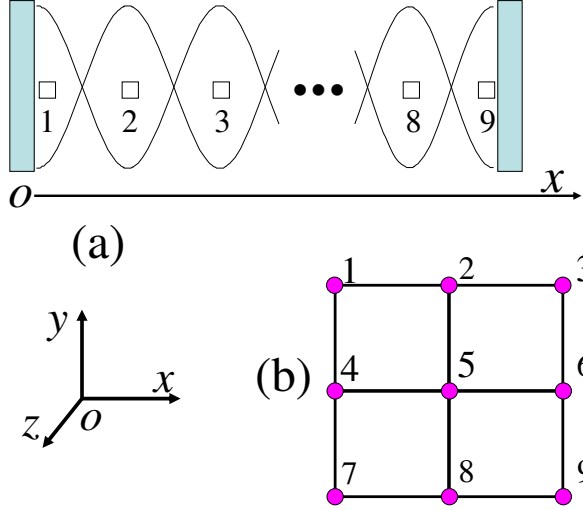


FIG. 2: (Color online) (a) Schematic superconducting devices (denoted as rectangles) array constructed along the cavity direction x with $N = 9$. The chosen coordinate is shown in the left-lower panel. Each device is placed parallel to the xoz plane and at the antinodes of the single-mode standing-wave cavity. The magnetic component of the cavity mode is along the y direction, which is perpendicular to the device loop plane, so that it is the only contributed component. Devices are placed at the antinodes of the cavity mode, so that the device-cavity interaction constant ($2g$) of different devices can be treated as the same constant. (b) An $N = 9$ linear array corresponds to a 2D 3×3 array, where the filled dots with the number represents the devices. For example, if one wants to generate the interaction in row 1, then devices 1, 2 and 3 are selected to achieve the J_x^2 coupling via the virtue excitation of the cavity mode, while the J_y^2 coupling among devices 2, 5 and 8 is implemented in order to achieve the interaction in column 2.

cavity-mediated interaction parts in these two SQUID loops considering that they can be made to be very small comparing with the inter-SQUID loop, or we can just put the magnetic shield to exclude the influence of the cavity mode magnetic flux in these two loops. The phase relation in the inter-SQUID loop is now modified as $\varphi_j^2 - \varphi_j^3 = 2\phi_j + 2g_j(a + a^\dagger) = 2\omega t + 2g(a + a^\dagger)$. Assuming all the devices work in their degeneracy points, under Lamb-Dicke (LD) limit and rotating-wave approximation as well as in the interaction picture with respect to $H_0 = \hbar\omega_c(a^\dagger a + \frac{1}{2})$, the cavity mediated interaction can be described by the Hamiltonian

$$H_{int} = i\hbar\beta \sum_{j=1}^N \sigma_j^\dagger (\eta a^\dagger e^{i\delta t} - \xi a e^{-i\delta t}) + \text{H.c.}, \quad (6)$$

where $gE_J/2\hbar = \beta \ll \delta = \omega_c - \omega \ll \omega$, $\sigma^\pm = \frac{1}{2}(\sigma^x \pm i\sigma^y)$, $\eta = e^{-i\phi_-} + e^{-i(2\phi_+ - \phi_-)}$, $\xi = e^{-i\phi_-} + e^{i(2\phi_+ + \phi_-)}$, and $\phi_\pm = \frac{1}{2}(\phi^1 \pm \phi^2)$. If $\phi_- = 0$ and $\phi_+ = k\pi$, then Eq. (6) reduces to

$$H_{int}^x = 2i\hbar\beta (a^\dagger e^{i\delta t} - a e^{-i\delta t}) J_x, \quad (7)$$

where $J_{x,y,z} = \sum_{j=1}^N \sigma_j^{x,y,z}$. The corresponding effective

Hamiltonian is given by [14, 15]

$$H_x = -\hbar\chi_x J_x^2, \quad (8)$$

where $\chi_x = 4\beta^2/\delta$. If $\phi_- = \pi/2$ and $\phi_+ = k\pi + \pi/2$, then Eq. (6) reduces to

$$H_{int}^y = 2i\hbar\beta (a^\dagger e^{i\delta t} - a e^{-i\delta t}) J_y, \quad (9)$$

with the effective Hamiltonian being given by

$$H_y = -\hbar\chi_y J_y^2, \quad (10)$$

where $\chi_y = \chi_x$. Actually the configuration described in Fig. (1a) is exactly equivalent to the device in Fig. (1b) in the specific case of $\phi^1 = \phi^2 \equiv 0$. So it is now seen that the required Hamiltonian (1) is unable to be achieved with the one SQUID in Fig. (1a) since only the interaction H_x could be realized.

It is clear that the specific choice of the dc magnetic flux can lead to the designated interaction type of the selected device. The cavity-device coupling and decoupling can be controlled by selecting the external magnetic flux to be dc or ac, not simply by changing the parameters of the device or the cavity. Since the external magnetic flux can be effectively controlled, the cavity-device interaction can be implement in the selected devices. In addition, all the devices are always stay in their degeneracy points in this implementation, where they possess long coherence time and minimal charge noises. Interestingly, the interactions in Eqs. (8) and (10), which correspond to the row and column interactions in Eq. (1), are insensitive to the thermal state of the cavity mode. It is notable that the evolutions governed by the Hamiltonians (8) and (10) may also be considered as unconventional geometric operations [11], which are robust against random operation errors [13], and thus have been essentially used in quantum information processing [5, 10, 11, 12, 15, 16].

With the two interactions in Eqs. (8) and (10), and given the fact that they can be implemented on selected devices, it is straight forward to implement the highly symmetrical Hamiltonian (1). The interaction exists only between the devices selected at one time, thus the spatial pattern of the physical array is not necessarily related to the 2D configuration directly. A possible configuration rendering the implementation of the Hamiltonian easier in existing systems consists of using a linear array of devices [7]. To implement a topologically protected logical qubit consists of $M \times M$ physical devices in 2D lattice, we can use a 1D linear array with $N = M^2$ devices, where the i th row and j th column in the 2D lattice correspond to the devices $(i-1) \times M + 1, (i-1) \times M + 2, \dots, i \times M$ and $j, j+M, \dots, j+(M-1) \times M$ in the 1D array, respectively. A small 3×3 2D array is illustrated in Fig (2b) as a example. After scaling the interactions of each row and column, the resulting Hamiltonian is the sum of the terms in Eq. (1). The coupling between rows and columns may be avoided [7] by applying operations that alternate between rows and columns provided that each operation time τ satisfies $\tau\chi_{x,y} \ll 1$.

Topologically protected qubits usually consist of many physical qubits and thus seem to be resource-consuming strategy. In fact, a small finite array, e.g., $M = 3$ for Hamiltonian (1) [7] and $M = 5$ for nearest neighbor model [8], are

likely sufficient to provide good protection from the noise by suppressing its effect to be many orders of magnitude lower, which could meet most practical demands.

One experimentally feasible procedure to generate a topologically protected qubit state is as follows. (i) Applying a large effective field along the x direction (putting $\bar{n} = 1/2$ and $\phi = 0$ or $\phi = \pi$) when devices are decoupled from the cavity, we get $H_1 = \pm E_J \sum \sigma_{i,j}^x$. Then the initial state is logical $|0\rangle = \prod |0\rangle_{ij}$ or logical $|1\rangle = \prod |1\rangle_{ij}$ depending on the overall sign in the Hamiltonian H_1 . (ii) Adiabatically switching off H_1 and then switching on the Hamiltonian (1). After performing these two steps, the two initial unprotected logical states $|0\rangle$ and $|1\rangle$ evolve into one of the protected ground states of the model Hamiltonian (1).

We now briefly address the experimental feasibility in implementing our scheme. Suppose that the quality factor of the superconducting cavity is $Q = 1 \times 10^6$. For the cavity mode with $\omega_c/2\pi = 50$ GHz [17], the cavity decay time is $\tau_c \approx 3.2 \mu\text{s}$. With a moderate vacuum Rabi frequency $\Omega \sim 10$ MHz and the lifetime of the SQUID $\gamma = 2 \mu\text{s}$, the strong-coupling limit can be readily reached [4]. Meanwhile, $2g = 10^{-2}$ [5] to ensure the LD limit. With $E_J = 40 \mu\text{eV}$ [5], $\beta/2\pi \approx 48$ MHz. To maintain the large-detuning condition, we can choose $\delta \sim 10\beta$, which in turn satisfies readily the requirement of $\delta \ll \omega \sim \omega_c$. A typical operation time for large-detuning interaction is $t \approx 10$ ns [5], which ensures that thousands of operations are possible [4] since the coherence time of the SQUID and cavity mode are both in the order of μs . When all devices are located at the antinodes of the microwave, the distance of the two neighbor devices is a half of the microwave wavelength. For the chosen cavity

mode ($\lambda_c = 6$ mm), 10 devices may be constructed along the cavity direction [17]. Typically, a SQUID loop ($1 \times 1 \mu\text{m}$) consists of two small identical JJs of the size $0.1 \times 0.1 \mu\text{m}$, and the inter-SQUID loop can be fabricated in the size of $4 \times 4 \mu\text{m}$. Therefore, the distance of the two devices is about 10^3 times of the device size, so that their mutual induction may be safely neglected. Moreover, it is notable that other kinds of 4-junction qubits have already fabricated and used in the experiments, *e.g.*, in [18]. Thus, the current design of qubit impose no extra requirements on the current technology of the qubit fabrication process. Finally, we roughly estimate the effect of the nonuniformity in device parameters, $E_{ce,j}$ and E_j^l , to the operations in our implementation. As different devices can be addressed and controlled individually, it is easy to tune each device in its degeneracy point via its gate voltage separately, such that the nonuniformity of E_{ce} among different devices is not of significant importance. The nonuniformity of the Josephson couplings with a deviation of ϵE_j will cause a minor change of the operation infidelity in the order of $\exp(-\Delta/\epsilon\chi_x)$, which is about 1% and thus can be neglected for the reasonable experimental parameters $\epsilon \sim 20\%$ and $\delta \sim 10\beta$, where $\Delta \sim \chi_x$ is the energy gap of topological phase.

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